Gravitational Lensing

Stephen Mullens 4000404
Department of Physics
University of Nottingham
Nottingham
NG7 2RD
United Kingdom

Abstract

Gravitational lensing is a term which has its origins in well known optical systems, in an optical lens the path of light changes because of the different refractive index of the lens material, in gravitational lensing it is gravity which causes the path of the light to change, not a change of medium. The goal of this project is to simulate gravitational lensing using computer models to try to better understand the gravitational lensing phenomena and justify real world observations. Ray tracing techniques will be used to simulate the movement of individual rays of light as they pass through massive objects. Many rays with randomly distributed different initial conditions can be examined and an image built up which represents what would be seen by an observer, using a random number approach is referred to as a Monte Carlo method. Here we show how observations such as Einstein Rings and Arc can be reproduced, and propose possible physical systems which could be responsible.

Introduction

A gravitational lens occurs because massive objects bend and distort space-time around them, light rays passing near follow the straightest possible worldline (path of least action) which causes them to change direction to an external observer.

As shown by Carroll and Ostlie (1996), the deviation in the angle in radians of a ray of light as it passes a massive object is:

$$\phi = \frac{4GM}{rc^2} \qquad \text{Eqn. 1}$$

Where r is the closest distance the ray passes from the centre of mass, M is the enclosed mass and all other symbols have their normal definitions. From this formula it is possible to analytically solve simple lensing situations with point mass lenses.

Initial Assumptions

Firstly, rays of light are considered to travel in straight lines, only being deflected by the lens at a single discontinuity in the plane of the lens and parallel to the observer. The magnitude of that deflection is a function of the distance from the lens at the point of deflection.

Secondly, the reciprocity of ray paths means that the path of a ray is independent of the direction of propagation along it. For this reason, we consider paths from the observer to the source. The gain in efficiency from this can be seen by consideration of a source object emitting light isotropically. Each ray has a vanishingly small probability of reaching the observer. By considering rays 'fired' from the observer, efficiency can be increased as time is not wasted following rays that can never be observed.

Clearly, it makes no sense to fire beams in all directions from the observer. Instead we limit them to a small angular range in the direction of the source, thus maximising efficiency. This also has physical significance because the observer, typically a telescope, can only take in light from a very limited range of angles. The range of angles used for this is calculated by consideration of the limiting case when a ray fired from the telescope will only just strike the edge of the source galaxy.

The final in-built assumption is that, when an angle is being used, small angle approximations can be used to simplify trigonometric functions. The angles of interest in gravitational lensing are of the order of a few arc seconds or 10^{-5} radians.

Monte Carlo Techniques

A Monte Carlo process is one which involves using random numbers to quickly find an approximate solution to a problem. In this instance, rays will be fired at random angles within the established parameters, an image of what the observer sees will eventually be built up. The level of detail in the image will be dependent on the number of these random rays used. The beauty of this process is that an image can be produced and if it is believed more rays will improve it, computationally it is a simple to just add more rays.

This approach may seem unnecessary when an easier approach could be to loop through ranges of ray angles. Looping through would result in a perfectly even distribution of ray angles but this lacks physical significance, in real systems the expectation is for emission to be randomly distributed. The expectation is for randomly distributed rays to converge faster and more physically towards a realistic lensed image.

The Einstein Ring

For the case where the observer, lens and source are perfectly in line, light rays emitted from the source are bent in a manner which produces a ring shape from the observers perspective.

Figure 1 shows the path of a ray which travels from the source and is bent just enough by the lens to strike the observer. If the path of the ray is traced back from the perspective of the observer then it will appear to come from the top point on the circle. The system illustrated is clearly cylindrically symmetric about the central axis. This means that the point produced by the single ray can be rotated around the axis to produce a ring. The ring in this case will have an angular radius of θ radians on the sky. This effect is known as an Einstein Ring,

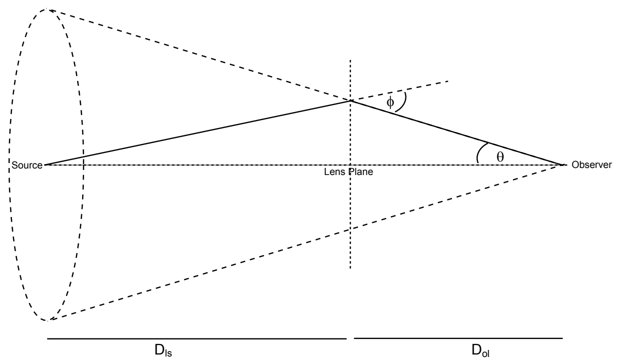

Fig 1 - Einstein Ring ray diagram

The rays contributing to the ring satisfy the condition

$$r = \theta D_{ol} = D_{ls}(\phi - \theta) \quad \text{Eqn. 2}$$

where r is the distance from the origin in the lens plane. This can then be rearranged to find θ and the formula for the angular deflection substituted in.

$$\theta = \sqrt{\left(\frac{D_{ls}}{D_{ol}}\right)\left(\frac{4GM}{c^2}\right)\frac{1}{(D_{ls}+D_{ol})}} \quad \text{Eqn. 3}$$

This formula shows that as the mass of the lens rises, it is possible to produce much larger rings, however even with massive objects such as black holes, the radius is only of the order of 10^{-5} radians.

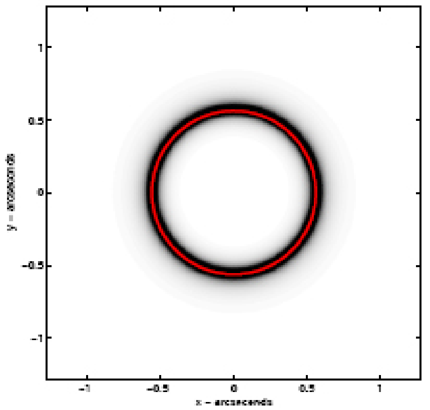

Fig 2 - Simulated Einstein Ring

Figure 2 shows a simulated Einstein ring image which was produced in matlab. The inner sharp ring is a plot of the theoretical Einstein ring from equation 3. The rings width is due to the finite size of the spherical source which produced it.

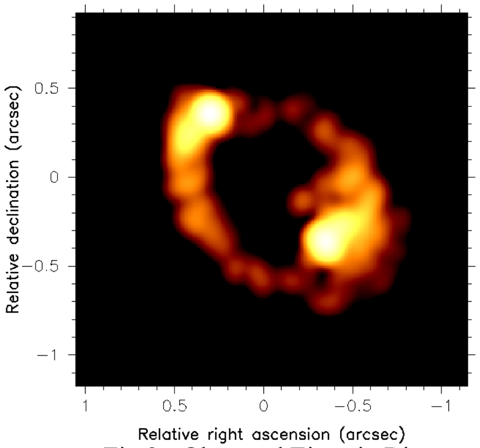

Fig 3 - Observed Einstein Ring

Figure 3 shows a real life Einstein ring phenomena, the ring in this case consists of two bright points and some less visible activity in an oval shape. The oval shape could be due to perspective but it is likely that the structure is at least slightly oval, this is because real life objects are never in perfect alignment and although they often have symmetries, they are not always spherically symmetric. The various textures and artifacts in the image are likely due to the lens being non point-like and existing as a mass distribution in three dimensions.

Off Axis Lenses

It now makes sense to consider cases where the lensing mass is slightly off axis. This is very physically significant because slightly off axis situations are far more likely than approximately perfect alignment.

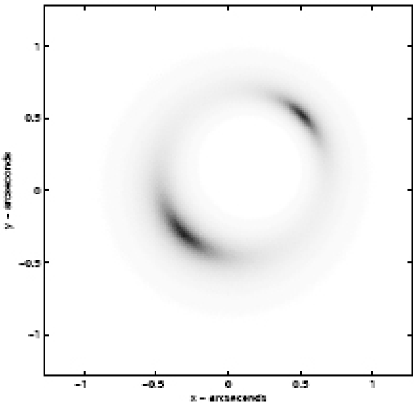

Fig 4 - Off Axis lensing - Elliptical source

Figure 4 shows an off axis point lens with a slightly elliptical source object, the result is clearly two point similar to figure 3, but it is clearly still circular and not an oval shape. The relative size of the two points can be adjusted by moving the lens more off axis. Obscuration of one half of this image would result in an arc, space dust could possibly account for this.

Non Point-like Lenses

A more physical two dimensional lensing mass is a step up towards reality and will clearly be required to produce images like the observed ring in figure 3.

A 128 by 128 pixel lens mass distribution can be constructed in either a graphical editing program or by using matlab. The mass in each pixel can be represented by a greyscale value, the total mass of the array of pixels should be normalised to a set value so that range of ray angles should stay roughly the same. This makes producing useable images much simpler.

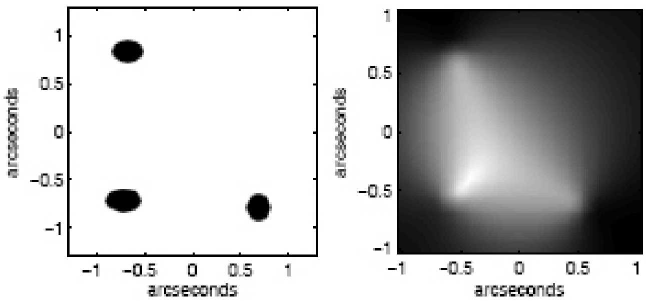

Fig 5 - Resultant mass felt

Figure 5 shows a mass distribution on the left and a resultant mass felt image on the right. Darkness represents the presence of mass / mass felt, i.e. the white space in the mass felt image is where a ray would feel relatively little mass.

The expectation is that all rays are lensed towards the centre of mass of the lens but can feel drastically different masses depending on where they pass through the lens plane.

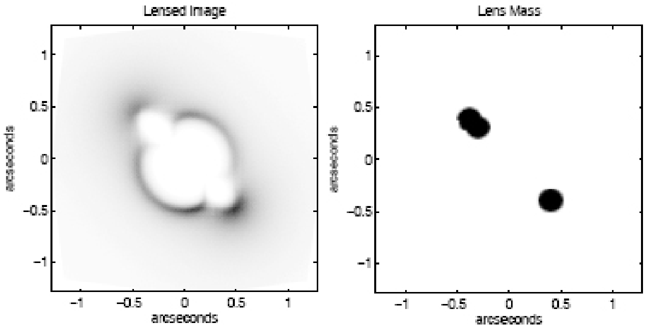

Fig 6 - Attempt to re-create observations

Figure 6 shows an attempt at reproducing the observation in figure 3. The lens mass in this case is slightly non physical but could be a two black holes orbiting one another. The lensed image does resemble figure 3 but it is by no means perfect and does not contain any fine detail. Fine detail is the result of extremely complicated lens mass distributions. In this case a 128 by 128 pixel mass distribution was used, to produce more realistic images a much larger array would be needed but this becomes exponentially more computationally expensive.

Conclusion

Reproduction of known observations is extremely difficult because you require detailed information about the lensing mass. Establishing the lens mass distribution is a complicated problem in itself but evidence can be gathered through a variety of astronomical observations and physical arguments.

Nevertheless the ray tracing method used here has produced meaningful results and clearly shown the fundamental physics behind observations.

References

Carroll & Ostilie,1996, An Introduction to Modern Astrophysics, Page 1205-1211, Addison Wesley Longman

Fig. 3 http://www.atnf.csiro.au/people/jlovell/